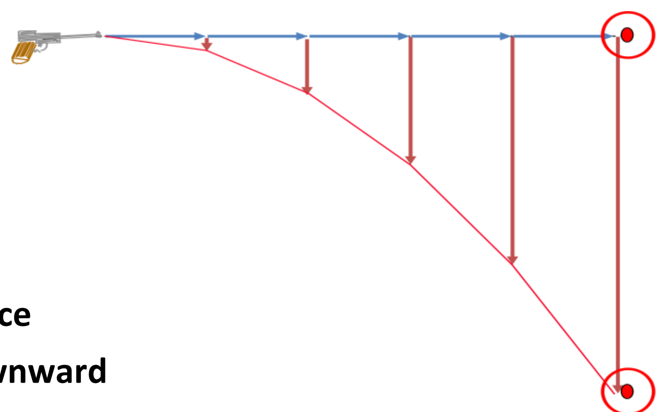
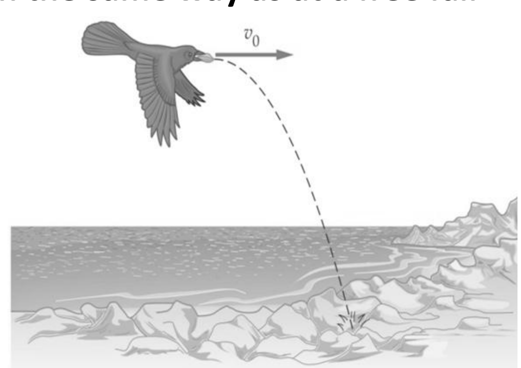
Chapter 7 Two-Dimensional motion

- projectile motion.
  - An object may move in both the *x* horizontal and *y* vertical directions simultaneously.
  - The form of two-dimensional motion we will deal with is called projectile motion.
- "A projectile is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity"
  - The free-fall (vertical) acceleration g is constant over the range of motion and directed down
  - The effect of air friction is negligible
  - With these assumptions, an object in projectile motion will follow a parabolic path in a vertical plane
  - Motion in the *x*- horizontal and *y*- vertical directions should be solved separately:
  - First we will look at a launch angle of zero

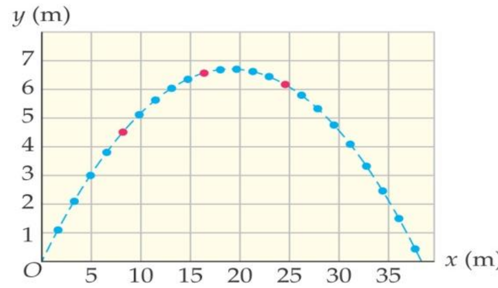


    - Zero Launch Angle
      - Assumptions:
      - ignore air resistance
      - $g = 9.81\ m/s^2$, downward
      - ignore Earth's rotation
      - If *y*-axis points upward, acceleration in *x*-direction is zero and acceleration in *y*-direction is $-9.8\ m/s^2$
    - The x-component of velocity remains the same!
    - The y-component of velocity changes in the same way as at a free fall motion

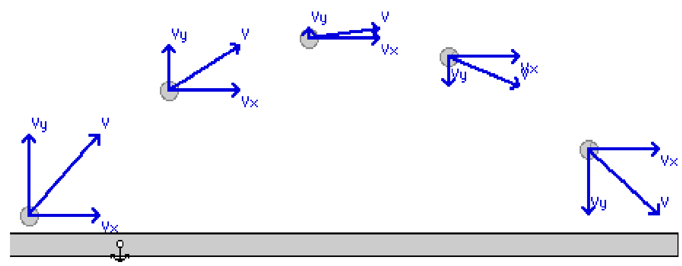


  - A crow is flying horizontally with a constant speed of 2.70 m/s when it releases a clam from its beak. The clam lands on the rocky beach 2.10 s later. Just before the clam lands, what is

- (a) its horizontal component of velocity, and
- (b) its vertical component of velocity?
- (c) How would your answers to parts (a) and (b) change if the speed of the crow were increased? Explain



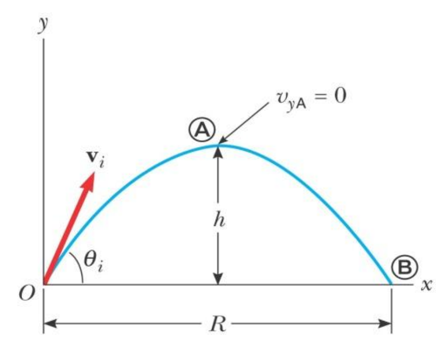
- **Projectiles with a launch angle above zero**
- **Analyzing Projectile Motion**
  - **Consider the motion as the superposition of the motions in the *x*- and *y*-directions**
  - **The *x*-direction has constant velocity**
  - ***Acceleration, a_x* = 0**



- **The y-direction is free fall**
  - ***Acceleration, a_y = -g***
  - **The *y*-component of the velocity is zero at the maximum height of the trajectory**
  - **The y acceleration stays the same throughout the trajectory**
- **The x acceleration stays at zero**
- **The initial velocity can be broken down into its x- and y-components:**
  - **Projectile Motion – Problem Solving Hints**

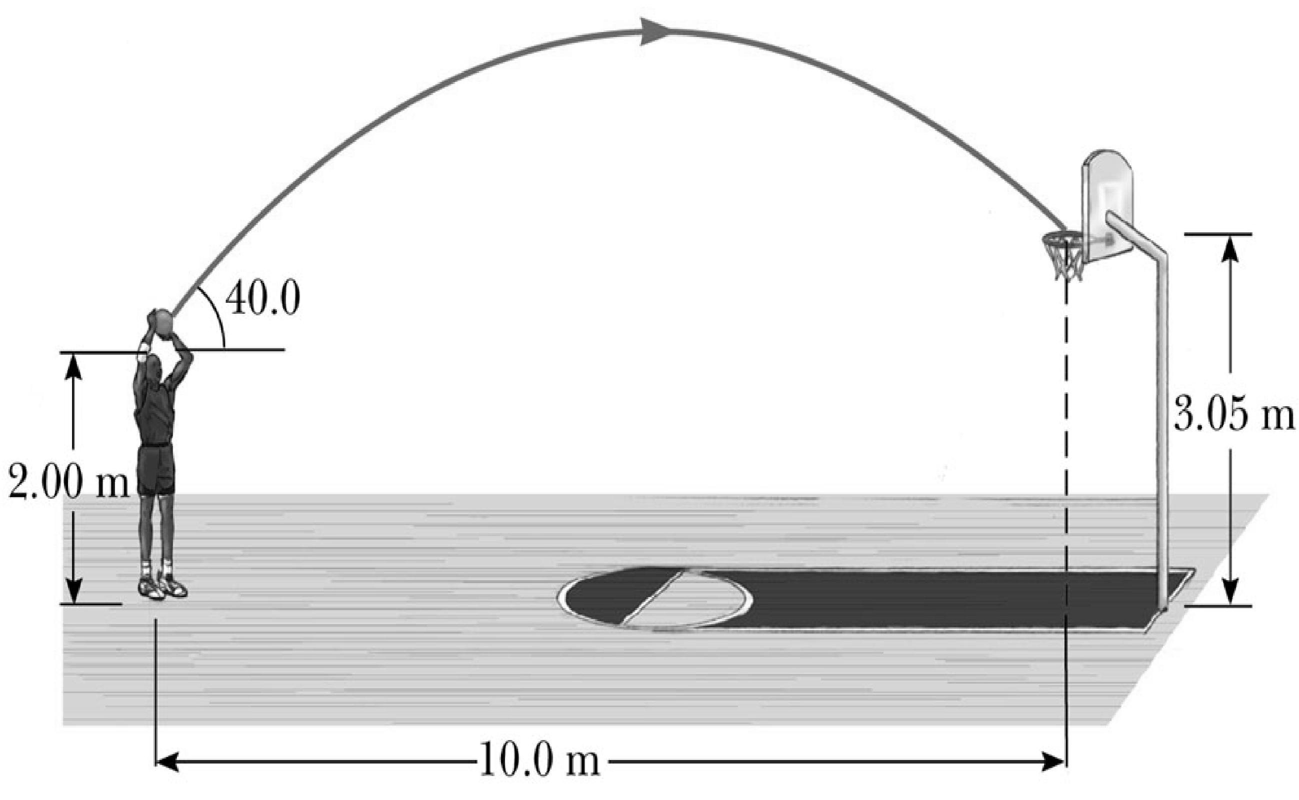
$$v_{Ox} = v_O \cos\theta_O \qquad v_{Oy} = v_O \sin\theta_O$$

    - **Select a coordinate system**
    - **Resolve the initial velocity into *x* and *y* components**
    - **Analyze the horizontal motion using constant velocity techniques**
    - **Analyze the vertical motion using constant acceleration techniques**
    - **Remember that both directions share the same time**

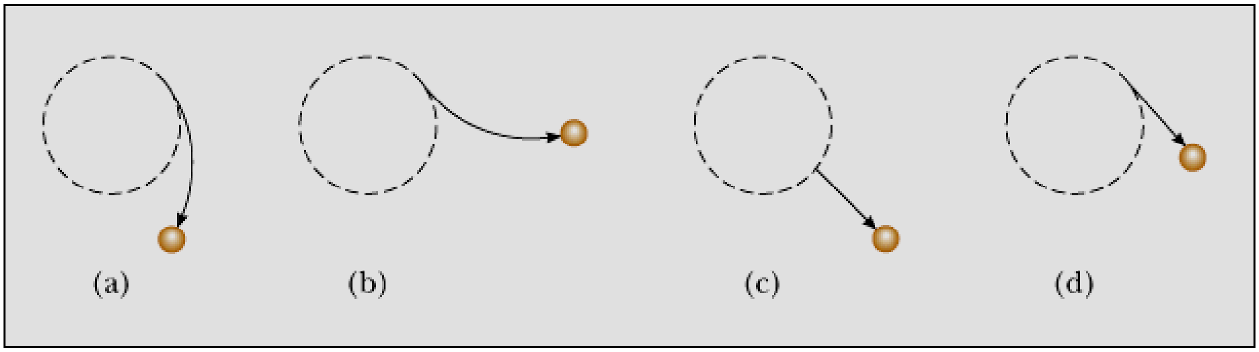


  - **Verifying the Parabolic Trajectory, cont**
    - **Displacements**
      - **$d_X = v_X t$**
      - **$v_x = (v_O \cos\theta)$ so $d_x = (v_O \cos\theta)\, t$**
      - **$d_Y = v_{Yi} t + 1/2 a_Y t^2 = (v_O \sin\theta) t - 1/2\, g t^2$, or**
      - **down time distance $d_y = ½\, g t^2$ where t is total time divided by 2**

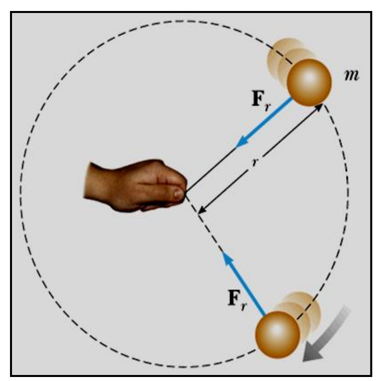
- **When going up and then down (y direction) the distance = 0, so use the equation** The maximum height the projectile reaches is *h*
  - **d^y = the vertical distance traveled is zero d^y=0**
  - **So $0 = v_i t + ½ at^2$**
  - **so, $- ½ at^2 = v_i t$**
  - **so then $t = -2v^y/g$**
    - **since $v^y = v_o (\sin θ)$**
    - **then $t = -2(v_o * \sin θ)/g$**
  - **Maximum Height of a Projectile use $vf^2=vi^2 + 2ad$**
    - **$h = (vf^2 - vi^2)/2g$, where vf at the top = 0**
    - **or, $h = (\sin θ \ v_0)^2 /2g$**
  - **to solve for distance on the x axis**
    - **$d_x = v_x t$ and $v_x = v_o \cos θ$**
    - **so $d_x =(v_o \cos θ)t$**
- **Range** - **from this information we build the range equation since $d_x =(v_o \cos θ)t$ and $t = -2(v_o * \sin θ)/g$**
  - **then $d_x =(v_o \cos θ)(-2(v_o * \sin θ)/g)$**
  - **From this point forward we will refer to d_x as range** (The range, *R*, is the horizontal distance of the projectile)
  - **The v_o is squared and the negative cancels out and the law of sins causes the sine and cosine to = sin 2θ**
  - **So Range = $(v_o^2 \sin 2θ)/g$**
  - The maximum range occurs at $q_i = 45^o$
  -



- Uniform Circular Motion
  - ***Uniform circular motion*** occurs when an object moves in a circular path with a constant speed
    - An acceleration exists since the *direction* of the motion is changing
    - The velocity vector is always tangent to the path of the object



  - A particle is moving in a circular path.
  - If the force on the particle would suddenly vanish (string cut) in which direction would the ball fly off?
- Circular motion is the result of acceleration
  - Acceleration is the result of a net force
  - $a = \Delta v/\Delta t$ & $F/m = a$
  - So circular motion is the result of a net force.
  - Finding Circular acceleration
  - Put the velocity vectors together
  - Proportionally the radius (r) vector is equal to the length to the vector from A to B
  - So A-B/r = 1
  - The vector V_1 is equal to V_2
  - So $\Delta v/v = 1$
  - So $\Delta v/v = AB/r$
  - Since from A to B is measuring distance and $d = v\Delta t$ you can substitute vt for AB
  - So $\Delta v/v = v\Delta t/r$
  - So $\Delta v/\Delta t = v^2/r$
  - Since $\Delta v/\Delta t = a$ then centripetal acceleration is equal to V^2/r
  - so
  - So $V^2/r = a_c$ *
  - Measuring Velocity and Force in a Circle



- Measuring Velocity
  - Since Velocity is equal to distance/time
  - And the distance around a circle is $2\pi r/t = v$
- Since $v^2/r = a_c$ and $2\pi r/t = v$
  - Substitute for v and
  - $(2\pi r/t)^2/r = a$, or $4\pi^2 r/t^2 = a_c$

- Thus far we have applied Newton's law, F = m*a to linear motion.
- Now we'll apply it to rotational motion
- Force causes Acceleration
  - F=ma so $F_c = m(4\pi^2 r/t^2)$
- In most cases, the string force not only has to provide the force required for circular motion, but also the force required to balance the gravitational force.
- Important consequences:
  - You can never swing an object with the string aligned with the horizontal plane.
  - When the speed increases, the acceleration increases up to the point that the force required for circular motion exceeds the maximum force that can be provided by the string.

Centripetal, Centrifugal Forces, continued

- The car is accelerated toward the center of the curve by a *centripetal* (center seeking) force
- In your reference frame of the car, you experience a "fake", or fictitious centrifugal "force"
  - Not a real force, just inertia relative to car's acceleration
- Centripetal force accelerates things toward the center of the circle
  - F = ma so $F_c = ma_c$ so $F_c = m(4\pi^2 r/t^2)$

$$\boxed{\sum F = m \cdot a_c = m \cdot \frac{v^2}{r}}$$

- Any point where the centripetal force is released the object will fly in a trajectory perpendicular to the force or a path tangent to the circle
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Period

- The *period*, *T*, is the time required for one complete revolution
- The speed of the particle would be the circumference of the circle of motion divided by the period
- Therefore, the period is

$$T \equiv \frac{2\pi r}{v}$$

- Earth revolves on its axis once per day
- Earth moves in (roughly) a circle about the sun
- What are the accelerations produced by these motions, and why don't we feel them?
- Velocity at equator: $2\pi r$ / (86,400 sec) = 463 m/s
- $v^2/r = 0.034\ \text{m/s}^2$
  - ~300 times weaker than gravity, which is 9.8 m/s^2
- Makes you feel lighter by 0.3% than if not rotating
- No rotation at north pole $\rightarrow$ no reduction in *g*
- If you weigh 150 pounds at north pole, you'll weigh 149.5 pounds at the equator
  - actually, effect is even more pronounced than this (by another half-pound) owing to stronger gravity at pole: earth's oblate shape is the reason for this
- The earth is also traveling in an orbit around the sun
  - $v = 30{,}000$ m/s, $r = 1.5\times10^{11}$ m $\rightarrow v^2/r = 0.006\ \text{m/s}^2$
  - but gravitational acceleration *on our bodies from the sun* is exactly this same amount.

- **in other words, the acceleration that makes the earth accelerate in a circular orbit also acts on us directly, causing us to want to follow the same path as earth**
- **this is to be contrasted with the car going around a curve, in which friction between pavement and tires applies a force on the car, but not on us directly, causing us to want to go straight**

- **another way to say this: we are in free-fall around the sun**